Towards the Reconstruction of Three-dimensional Sub-wavelength Objects imaged by a Veselago-Pendry Superlens

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Abstract — A procedure to reconstruct a discrete approximation of a source object with finite depth from its image generated by a Veselago-Pendry superlens, is presented. It is validated by considering the simple case of an array of discrete field sources placed along a line normal to the lens.

I. INTRODUCTION

The Veselago-Pendry superlens can image arbitrary field distributions in a plane parallel to the superlens with subwavelength accuracy [1]. Such scenarios, typical in photolithography, have been discussed in the literature [2],[3]. However, little information is available on the superlens imaging and reconstruction of field source distributions which possess a finite depth (normal to the lens surface). In this paper we derive an algorithm for reconstructing a field source distribution that extends in normal direction.

There exist a number of inverse scattering algorithms that aim at reconstructing an unknown source distribution from far-field information collected in the space surrounding the source. The superlens image reconstruction is different from these situations in two respects. Firstly, the image contains both near- and far-field information by virtue of the evanescent amplification of the superlens. Secondly, the image field is usually sampled in a plane rather than on a closed surface surrounding the sources.

Note that we will not address the problem of realizing a superlens with negative refractive index $n = -1$ and free space wave impedance η_0. We simply consider that the lens has the ideal properties assumed by Veselago [1] and Pendry [2] in their classical papers, and that all its properties are linear. Reality effects can be introduced at a later stage by an appropriate modification of the spatial Fourier spectrum of the image [4].

In this letter we discuss the simple case of a monochromatic source field distribution located on a line normal to the lens (see Fig. 1). To formulate a discrete reconstruction algorithm we assume that the source field distribution is discretized into square elements or pixels, within which the electric field is constant and has an amplitude coefficient a_m, where m is the index of the source elements of size *dx'* x *dy'*. We further assume, for simplicity, that all elements of the source distribution are in phase. The algorithm then reconstructs the coefficients a_m of the discrete source distribution from its field in the image plane. We first compute the resulting image field using spatial Fourier decomposition of the source fields in *y*-direction and transferring each spectral component to the image plane using the transfer function of the waveguide model [4], then reconstruct the source coefficients using the inverse algorithm. The result should ideally yield the original source distribution.

II. MATHEMATICAL FORMULATION

Fig. 1 shows the geometry of a superlens of thickness *d* and the positions of the source object and the image plane. The lens of thickness *d* can potentially image objects up to a distance *d* behind it. For simplicity, and without loss of generality, we first assume that the source distribution, shown in Fig. 1, is uniform in the *z*-direction and periodically repeated in the transverse direction (*y*-direction) with a periodicity *s*. This allows us to treat the problem in terms of an equivalent one-dimensional spectral waveguide model [4]. The distance *s* is chosen to be larger than the extent of the object and the line of discrete source elements is assumed to be inside the *s* x *d* box.

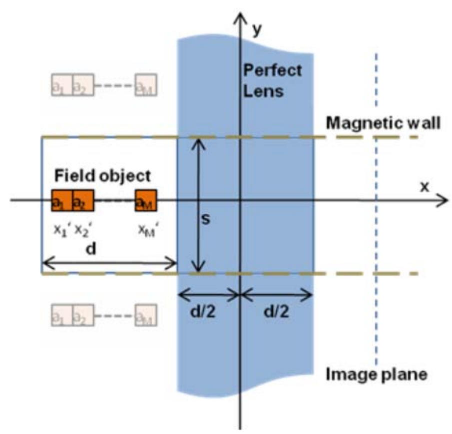

Fig. 1. Superlens imaging of a discretized source distribution with finite extent normal to the lens. The structure has a periodicity *s* in y-direction and is independent of z. We want to recover the amplitude coefficients of the source distribution from the field recorded on the image side.

The origin of the field and source coordinates is located in the center of the superlens slab. The steady-state electric field in the half-space $x>d/2$ can be expressed as the sum of the weighted discrete Green's functions $g_m(x-x_m', y-y')$, which are the electric fields produced by monochromatic unit field sources located at x_m', y'. The total image field in the half-space $x>d/2$ due to the array of sources is given by

$$E(x,y) = \sum_{m=1}^{M} a_m g_m\left(x - x_m', y - y'\right) \tag{1}$$

where a_m are the amplitude coefficients of the field source elements. The Green's function g_m is the sum of the spatial

Fourier components of the m-th source transmitted by the superlens; it can be computed either analytically or numerically. The numerically computed Green's functions will be approximations of their ideal counterparts, but will be more realistic since they contain only the first few spectral terms.

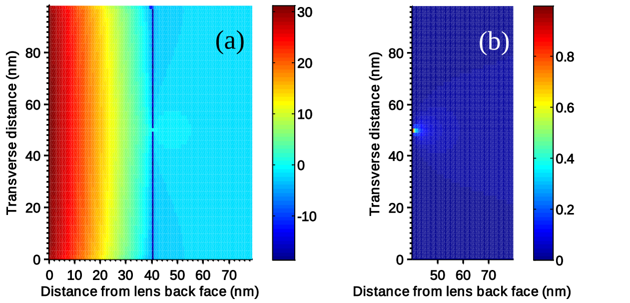

Fig. 2. Magnitude of the Green's function g in the half space x > d/2 for a pixel located at *d/2* behind the front face of the lens and at the center of the unit cell on the *y*-axis. (a) shows the amplitudes plotted on a log scale and (b) is plotted on a linear scale.

Fig. 2 shows the Green's function for a single source pixel located at *x'=-d* (*d/2* behind the lens front surface). The free-space wavelength emitted by the source is *400 nm* (*f = 750 THz*). The lens has a thickness *d = 80nm* (a period of *s = 100nm* = $\lambda_0/4$ has been chosen arbitrarily). The ideal Veselago-Pendry lens has the property that it can pass with full fidelity all the spatial harmonics of a given object field [4]. The magnitude of the image field strongly depends on the distance of the source from the lens. The field produced by a source farther from the lens is dwarfed by that produced by a source closer to the lens.

To determine the unknown source coefficients in the expression for the total field (1) we need to solve the inverse problem. Knowing the Green's functions produced by unit field sources, we can recover the unknown coefficients a_m by projecting the known image field function *E(x,y)* onto the manifold of the known Green's functions which we consider to be the basis functions of the function space. Let ***O*** represent the vector of projections of *E(x,y)* onto the set of Green's functions with *m = 1,2…M*. Each projection component O_m is the inner product of *E(x,y)* and g_m :

$$O_m = \langle E(x,y), g_m \rangle \qquad (2)$$

If we express *E(x,y)* by (1) and write the expression for the vector of projections in matrix form, we obtain $O = G \cdot A$ where *G* is an *M x M* square matrix with elements $G_{ij} = \langle g_i, g_j \rangle$, *i = 1..M* and *j= 1..M*, and ***A*** is the vector of unknown coefficients a_m which can be obtained by inverting the matrix ***G***.

III. Results and Discussions

Consider the series of discrete source elements shown in Fig. 1. The image field is generated and then the inversion procedure is used to assess the accuracy of the scheme. While calculating the Green's functions and the inversion, we note that it is not essential to sample the image field in the entire volume of the image space. It is sufficient to sample it only in a single plane parallel to the lens, since the field on the image side can be reconstructed from it, provided that both magnitude and phase of the spectral components are known in that plane.

The inversion procedure was implemented in Mathematica. A double precision and pseudoinversion was insufficient due to the high condition number of the Green's matrix. However, switching to a higher precision Green's matrix yielded satisfactory results. In Fig. 3 a series of random amplitude sources lying on a line normal to the lens was used to generate an image. The image at *d/2* behind the lens was then inverted to recover the amplitudes of the source elements. By moving the sampling plane closer to the lens, the accuracy of the inversion could be increased. In Fig. 4 the source amplitude is sinusoidal.

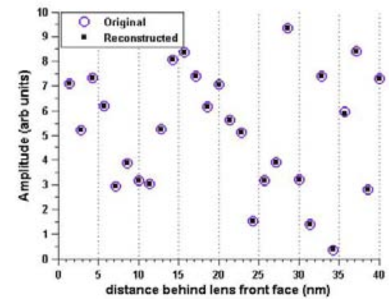

Fig. 3. The recovered 28 sources of random amplitude spread evenly on a line normal to the lens over the distance *–d/2<x'<-d*.

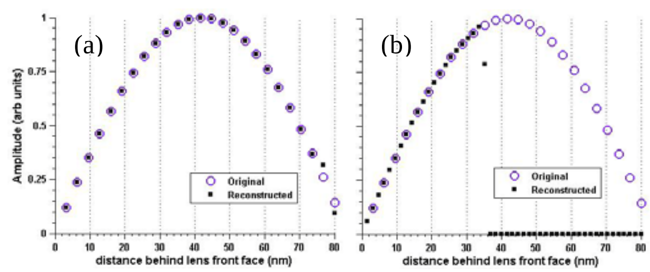

Fig.4. The recovered sources having a sinusoidal amplitude distribution along the x-axis, located between *–d/2<x'<-3d/2* for 25 and 50 sources.

IV. Conclusion

We have derived and tested an inversion algorithm that reconstructs a source distribution with a finite extent normal to a superlens from its image. It yields acceptable results for source distributions that extend up to half the lens thickness from its surface. These results suggest that it may be possible to reconstruct sub-wavelength three-dimensional objects from their superlens image.

V. References

[1] J. B. Pendry," Negative refraction makes a perfect lens," *Phys. Rev. Lett.*, vol. 85(18), pp. 3966-3969, Oct. 2000.

[2] V. G. Veselago," Remarks on electrodynamics of materials with negative refraction: nonmagnetic approach and superlens as a matching device," In *Proc. SPIE*, vol. 5955, pp. 595501, 2005.

[3] L. Solymar and E. Shamonina, *Waves in Metamaterials*, Oxford University Press, USA: Mar. 2009.

[4] R. S. Hegde et al, "Shedding new light on super-resolution imaging: a spectral domain approach", In *Fourth International Congress on Advanced Electromagnetic Materials in Microwaves and Optics*, Karlsruhe, Germany, Sep 2010.